

FIG. 1

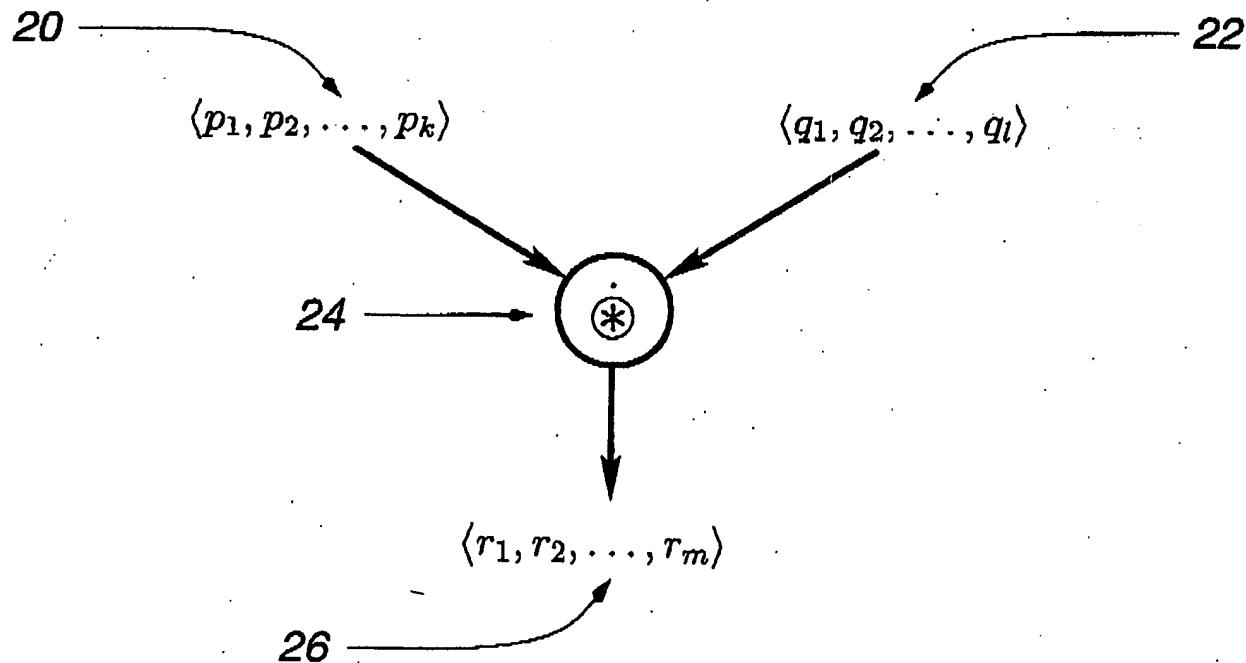


FIG. 2

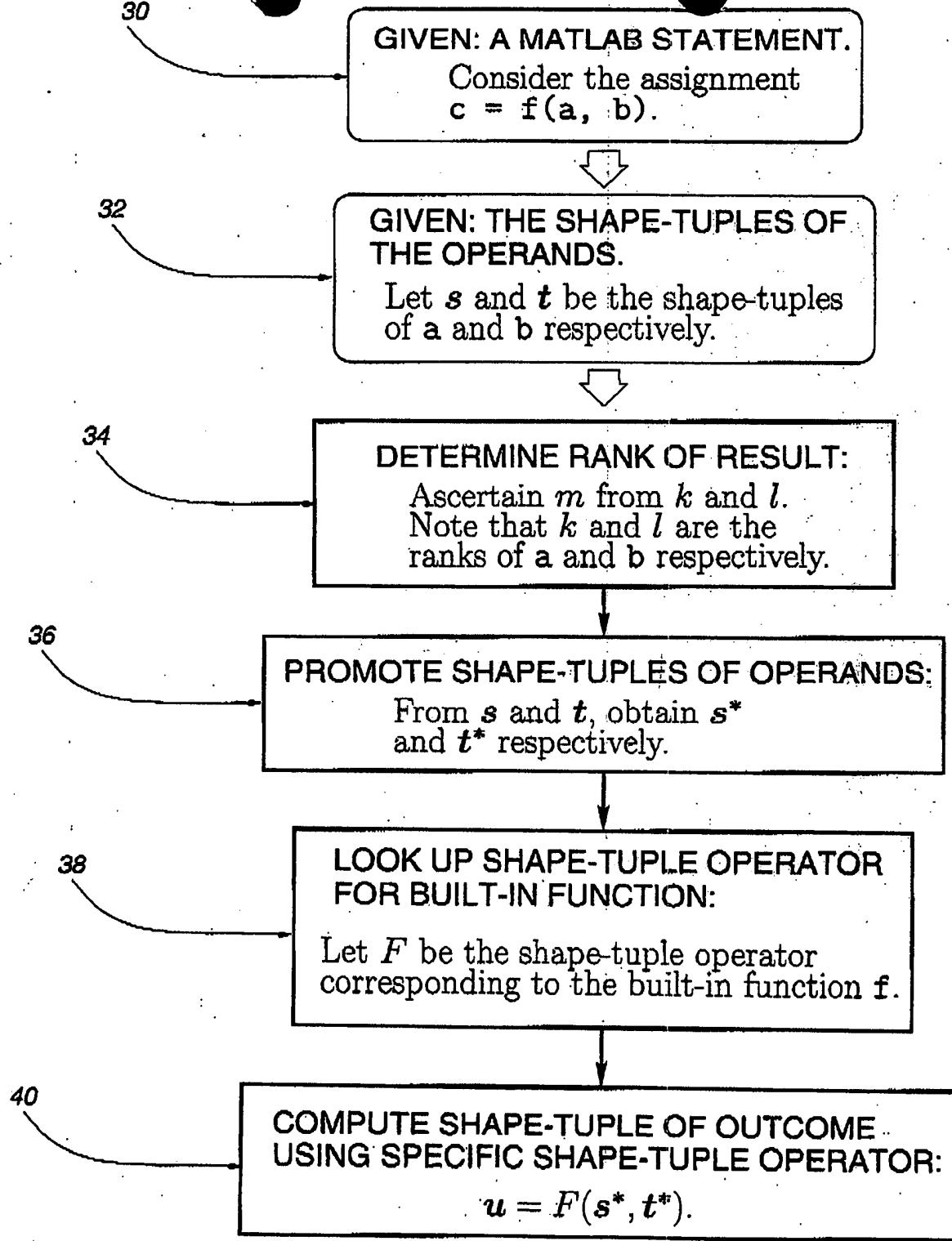


FIG. 3

| Operator                               | Rank               |
|--|--------------------|
| $a*b$                                  |                    |
| $a+b$                                  |                    |
| $a-b$                                  |                    |
| $a.*b$                                 |                    |
| $a.^b$                                 |                    |
| $a./b$                                 |                    |
| $a.\backslash b$                       |                    |
| $a==b$                                 |                    |
| $a\tilde{=}b$                          |                    |
| $a<b$                                  | $\max(R(a), R(b))$ |
| $a>b$                                  |                    |
| $a\leq b$                              |                    |
| $a\geq b$                              |                    |
| $a\&b$                                 |                    |
| $a b$                                  |                    |
| $a/b$                                  |                    |
| $a\backslash b$                        |                    |
| $[a, b]$                               |                    |
| $[a; b]$                               |                    |
| $+a$                                   |                    |
| $-a$                                   |                    |
| $\tilde{a}$                            | $R(a)$             |
| $a'$                                   |                    |
| $a.'$                                  |                    |
| $c(:) \leftarrow a$                    | $R(c)$             |
| $a\tilde{=}b$                          |                    |
| $a(:)$                                 | 2                  |
| $\text{rand}$                          |                    |
| $a:b$                                  |                    |
| $\text{rand}(a, b)$                    | 2                  |
| $\text{ones}(a, b)$                    |                    |
| $a(e)$                                 | $R(e)$             |
| $a(e_1, e_2, \dots, e_n)$              |                    |
| $\text{rand}(e_1, e_2, \dots, e_n)$    | $n$                |
| $\text{ones}(e_1, e_2, \dots, e_n)$    |                    |
| $c(e) \leftarrow a$                    | $\max(R(a), R(c))$ |
| $c(e_1, e_2, \dots, e_n) \leftarrow a$ | $\max(n, R(c))$    |

FIG. 4

| MATLAB Expression<br>e  | Shape Expression    | $\bar{\theta}(e)$  | $u$  |
|---|---------------------|--|--|
| $a+b$   | $s \dot{\oplus} t$  | $\bar{\theta}(a)\bar{\theta}(b)(1 - (1 - \bar{\alpha}(a))(1 - \bar{\alpha}(b))(1 - \bar{\beta}(a)\bar{\beta}(b)\delta(\Psi s\Psi\Gamma_1 - t\Gamma_1)))$                                 | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^*\bar{\alpha}(b) + t^*\bar{\alpha}(a)(1 - \bar{\alpha}(b)) + (s^*\Gamma_1 + t^*\Gamma_2 + I - \Gamma_1 - \Gamma_2)(1 - \bar{\alpha}(a))(1 - \bar{\alpha}(b)))$ |
| $a-b$<br>$a.*b$<br>$a.^b$<br>$a./b$<br>$a.\backslash b$<br>$a==b$<br>$a~=b$<br>$a< b$<br>$a> b$<br>$a<=b$<br>$a>=b$<br>$a&b$<br>$a b$ | $s \dot{\oplus} t$  | $\bar{\theta}(a)\bar{\theta}(b)(1 - (1 - \bar{\alpha}(a))(1 - \bar{\alpha}(b))(1 - \delta(s - t)))$  | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^*\bar{\alpha}(b) + t^*(1 - \bar{\alpha}(b)))$  |
| $+a$<br>$-a$<br>$~a$  | $i s$               | $\bar{\theta}(a)$  | $s^*$  |
| $a^\sim b$  | $s \dot{\oplus} t$  | $\bar{\theta}(a)\bar{\theta}(b)(1 - (1 - \bar{\alpha}(a)\bar{\beta}(b))\delta(t\Gamma_1 - \Psi t\Psi\Gamma_1))(1 - \bar{\alpha}(b)\bar{\beta}(a)\delta(s\Gamma_1 - \Psi s\Psi\Gamma_1))$ | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^*\bar{\alpha}(b) + t^*(1 - \bar{\alpha}(b)))$  |
| $a'$<br>$a.:$   | $\div s$            | $\bar{\beta}(a)$   | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)\Psi s^*\Psi$   |
| $a/b$   | $s \dot{\otimes} t$ | $\bar{\theta}(a)\bar{\theta}(b)(1 - \bar{\alpha}(b))(1 - \bar{\alpha}(a))(1 - \bar{\beta}(b))(1 - \bar{\beta}(a)\bar{\beta}(b)\delta(s\Gamma_2 - t\Gamma_2))$                            | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^*\bar{\alpha}(b) + t^*(1 - \beta(b)) + (s^*\Gamma_1 + I - \Gamma_1 - \Gamma_2 + \Psi t^*\Psi\Gamma_2)(1 - \bar{\alpha}(b))\beta(b))$                           |
| $a\backslash b$   | $s \dot{\otimes} t$ | $\bar{\theta}(a)\bar{\theta}(b)(1 - \bar{\alpha}(a))(1 - \bar{\alpha}(b))(1 - \bar{\beta}(a))(1 - \bar{\beta}(a)\bar{\beta}(b)\delta(s\Gamma_1 - t\Gamma_1))$                            | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(t^*\bar{\alpha}(a) + s^*(1 - \beta(a)) + (\Psi s^*\Psi\Gamma_1 + I - \Gamma_1 - \Gamma_2 + t^*\Gamma_2)(1 - \bar{\alpha}(a))\beta(a))$                           |
| $[a; b]$  | $s \dot{\oplus} t$  | $\bar{\theta}(a)\bar{\theta}(b)\delta(s(I - \Gamma_1) - t(I - \Gamma_1))$  | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^* + t^*\Gamma_1)$  |
| $[a, b]$  | $s \dot{\ominus} t$ | $\bar{\theta}(a)\bar{\theta}(b)\delta(s(I - \Gamma_2) - t(I - \Gamma_2))$  | $(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^*\Gamma_2 + t^*)$  |

FIG. 5

| Shape-Tuple Class Operator | Identity | Associativity | Commutativity | Idempotent Law |
|----------------------------|----------|---------------|---------------|----------------|
| $\oplus$                   | $i$      | $\times$      | $\times$      | $\times$       |
| $\oplus$                   | $i$      | $\checkmark$  | $\checkmark$  | $\checkmark$   |
| $\wr$                      | -        | -             | -             | -              |
| $\odot$                    | $i$      | $\checkmark$  | $\checkmark$  | $\times$       |
| $\Gamma$                   | -        | -             | -             | -              |
| $\oslash$                  | $\times$ | $\times$      | $\times$      | $\times$       |
| $\circ$                    | $\times$ | $\times$      | $\times$      | $\times$       |
| $\odot$                    | $\times$ | $\checkmark$  | $\checkmark$  | $\times$       |
| $\ominus$                  | $\times$ | $\checkmark$  | $\checkmark$  | $\times$       |

FIG. 6